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Abstract

The hypercharges of the fermions are not uniquely determined in SO(10) grand unification, but rather depend upon which linear combination of the two U(1) subgroups of SO(10)$\supset$SU(3)$\times$SU(2)$\times$U(1)$\times$U(1) remains unbroken. We show that, in general, a given hypercharge assignment can be obtained only with very high-dimensional Higgs representations. The observation that the standard model is obtained with low-dimensional Higgs representations can therefore be regarded as further evidence for SO(10) grand unification. This evidence is independent of the fact that SO(10)$\supset$SU(5).

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The standard model of the fundamental interactions is based on the gauge group $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. Each generation of fermions transforms as a reducible representation of the gauge group, consisting of five irreducible representations.[2] The hypercharges of the representations are chosen to reproduce the observed electric charges of the fermions.

One of the most compelling pieces of evidence for grand unification is that the fermions of each generation transform as the $\overline{5}$ + 10-dimensional representation of SU(5) [1]. The five irreducible representations of each generation of fermions are thereby unified into two, and the three gauge groups are unified into one. The hypercharges of the five irreducible representations are uniquely determined by their embedding in the $\overline{5}$ + 10 of SU(5).

If a right-handed neutrino exists, the group-theoretic evidence for grand unification is even more compelling: the fermions of each generation transform as the 16-dimensional representation of SO(10) [2, 3]. The six irreducible representations are thereby unified into a single irreducible representation, and the three gauge groups are unified into one. We assume the existence of a right-handed neutrino for the remainder of the discussion.

SO(10) has a subgroup $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$. When SO(10) is spontaneously broken to $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$, the hypercharge subgroup[3] is a linear combination of the two U(1) subgroups of SO(10). Thus the hypercharges of the fermions are not uniquely determined in SO(10) grand unification, in contrast to the case of SU(5) grand unification, but rather depend upon which linear combination of the two U(1) subgroups is unbroken.

The $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ quantum numbers of the left-handed fields which make up the 16-dimensional representation of SO(10) are given in Table 1. The hypercharge is normalized such that the left-handed positron has unit hypercharge. The parameter a depends upon which linear combination of the two U(1) subgroups is unbroken. It is a rational number because the hypercharges are "quantized", i.e., commensurate, since a U(1) subgroup of a non-Abelian group is necessarily compact [1].[4]

The value of the parameter a depends upon the Higgs representation employed to break SO(10) to $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. The Higgs field may be either fundamental or composite; only its group-theoretic properties are relevant to the considerations of this paper. The candidate values of a for a given irreducible representation correspond to the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ singlets contained in that representation [9]. Usually this representation must be accompanied by at least one additional Higgs irreducible representation in order to break SO(10) down to $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, because the latter is generally not a maximal little group of the former for a single irreducible representation [9]. To generate fermion masses, the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ symmetry must be broken by yet one or more additional Higgs irreducible representation, chosen from the 10-, 120-, and 126-dimensional representations (since $16\times 16 = 10 + 120 + 126$). The $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ symmetry is broken to $\mathrm{U}(1)_{EM}$

[2]This does not include a right-handed neutrino. We shall introduce this particle into the discussion shortly.

[3]The hypercharge subgroup is, by definition, the unbroken U(1) subgroup. In general, this does not correspond to the usual hypercharge subgroup of the standard model.

[4]This representation is free of gauge and gravitational anomalies for any value of a, not just for rational values [4, 5, 6, 7]. It thus serves as an example of a chiral, anomaly-free gauge theory that (for irrational values of a) cannot be embedded in a grand-unified theory [8].

Table 1: Quantum numbers of the left-handed fields which make up the 16-dimensional representation of SO(10). The parameter a depends upon how the $\mathrm{U}(1)_Y$ subgroup is embedded in SO(10).

	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{EM}$
(u_L, d_L)	3	2	a	$(1-2a, 4a-1)$
u_L^c	$\overline{3}$	1	$2a-1$	$2a-1$
d_L^c	$\overline{3}$	1	$1-4a$	$1-4a$
(ν_L, e_L)	1	2	$-3a$	$(1-6a, -1)$
ν_L^c	1	1	$6a-1$	$6a-1$
e_L^c	1	1	1	1

when any of the color-singlet, SU(2) doublets contained in these representations acquires a vacuum-expectation value, leading to the electric charges listed in the last column of Table 1. The standard model evidently corresponds to $a = 1/6$.

We have shown by construction that any rational value of a can be obtained by an appropriate choice of the Higgs irreducible representation. However, a given value of a generally requires a very large Higgs irreducible representation. In practice, the smallest Higgs irreducible representations yield only a few values of a. We list in Table 2 the possible values of a, and the Higgs irreducible representations which can yield that value, for all Higgs representations of dimension 55440 or less.[5] Note that a and $a/(6a-1)$ are equivalent, upon interchanging u_L^c with d_L^c and ν_L^c with e_L^c. Thus we list values of a in the interval $[0, 1/3]$ only. Higgs representations listed as "undetermined" have SU(3)×SU(2)×U(1)×U(1) singlets, which do not determine a. Higgs representations listed as "none" have no SU(3)×SU(2)×U(1) singlets. If one or more additional Higgs irreducible representations are needed to break SO(10) to SU(3)×SU(2)×U(1), as is generally the case, they must correspond to the same value of a or an undetermined value of a.

It is satisfying that the standard model ($a = 1/6$) is obtained with several small Higgs irreducible representations,[6] the 16-, 126-, and 144-dimensional representations, as is well known [11]. If we lived in a world in which the ratio of the hypercharges of the quark doublet and the positron were, say, 1/8 rather than 1/6, we could still embed the fermions in the 16-dimensional representation of SO(10), but we would need a 9504-dimensional Higgs representation to obtain the desired symmetry breaking. While there is (perhaps) nothing

[5] These were derived with the help of the tables of Refs. [9, 10] and the *Liegroup* package developed by George M. Hockney.

[6] Generally accompanied by at least one other irreducible representation, such as the 45-dimensional representation [11].

fundamentally wrong with this, it is less palatable than a model which requires only Higgs fields in low-dimensional irreducible representations. These results are independent of the fact that SO(10)$\supset$SU(5); for example the 144-dimensional representation, which contains no SU(5) singlet, produces $a = 1/6$, while the 210-dimensional representation, which does contain an SU(5) singlet, produces $a = 1/3$.

We believe that the economy of the Higgs representation in SO(10) grand unification, while well known, has not been fully appreciated. We regard it as further evidence for SO(10) grand unification.

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Table 2: Values of the parameter a corresponding to the SU(3)×SU(2)×U(1) singlets of SO(10) Higgs representations up to dimension 55440. The standard model corresponds to $a = 1/6$. The representations labeled "undetermined" have SU(3)×SU(2)×U(1)×U(1) singlets, and those labeled "none" have no SU(3)×SU(2)×U(1) singlets.

a	SO(10) Higgs representation
1/6	16, 126, 144, 560, 672, 720, 1200, 1440, 1728, 2640, 2772, 2970, 3696, 3696′, 4950, 5280, 6930′, 7920, 8064, 8800, 9504, 10560, 11088, 15120, 17280, 20592, 20790, 23760, 25200, 26400, 27720, 28160, 28314, 29568, 30800, 34398, 34992, 36750, 38016, 39600, 43680, 46800, 48048, 48048′, 48114, 49280, 50050, 50688, 55440
1/3	45, 210, 770, 945, 1050, 1386, 4125, 5940, 6930, 7644, 8085, 8910, 12870, 14784, 16380, 17325, 17920, 23040, 50688, 52920
0	120, 126, 1728, 2772, 2970, 3696′, 4125, 4312, 4950, 6930, 6930′, 10560, 20790, 27720, 28160, 28314, 34398, 36750, 42120, 46800, 48114, 50050, 50688
1/4	560, 1440, 3696, 5280, 8064, 8800, 11088, 15120, 23760, 25200, 29568, 30800, 34992, 38016, 39600, 43680, 46800, 48048, 49280, 55440
1/12	672, 1200, 8800, 11088, 17280, 23760, 25200, 26400, 28314, 30800, 34992, 38016, 49280, 55440
1/9	2772, 6930, 50688
2/9	3696′, 6930′, 20790, 34398, 36750, 46800, 48114
5/18	8064, 34992, 39600, 43680
1/8	9504, 29568
1/18	9504, 29568, 30800
5/24	17280, 26400
2/15	28314
undetermined	45, 54, 210, 660, 770, 945, 1050, 1386, 4125, 4290, 5940, 6930, 7644, 8085, 8910, 12870, 14784, 16380, 17325, 17920, 19305, 23040, 50688, 52920
none	10, 210′, 320, 1782, 4410, 4608, 9438, 31680, 37180, 37632, 48510